Homework Solution SET #1

Homework 1: Problems 2.1-2.6 at the end of Chapter 2 in the Nilsson textbook
Due Tuesday September 1, 2009, Lecture 4, in class

2.1 Write the following Boolean function in DNF:

$$f = (x_1 + x_2)(x_3 + x_4)$$

$$f = x_1x_3 + x_2x_3 + x_1x_4 + x_2x_4$$

2.2 Show that

$$x_1x_2x_3 + \bar{x}_1x_2x_3 = x_2x_3$$

Factoring yields:

$$x_1x_2x_3 + \overline{x_1}x_2x_3 = (x_1 + \overline{x_1})x_2x_3.$$

Then, $(x_1 + \overline{x_1})$ can be replaced by 1.

2.3 Indicate which of the following Boolean functions of three input variables can be realized by a single threshold element with weighted connections to the inputs. You do *not* need to calculate the weight and threshold values:

1. x_1
2. $x_1x_2x_3$
3. $x_1 + x_2 + x_3$
4. $(x_1x_2x_3) + (\bar{x}_1\bar{x}_2\bar{x}_3)$
5. 1

The first three and the last functions are realizable by a single threshold element (that is, they are linearly separable), and the fourth one is not. These results can be easily visualized by making a sketch of a 3-d cube and noting those vertices for which the functions have value 1.

2.4 Prove that there are exactly 3^n monomials of n dimensions and 3^n clauses of n dimensions.

Because clauses are duals of monomials (that is, each clause is the negation of some monomial and each monomial is the negation of some clause), there are also 3^n clauses. An atom x_i occurs in a monomial positively, negatively, or not at all. Because there are three mutually exclusive possibilities for each of the n atoms, there are 3^n monomials.

2.5 Refer to the definitions of the features, x_1, x_2, x_3, x_4 on page 24 and to the rules for action on page 25. Show that the assumption that there are no "tight spaces" in the two-dimensional grid world implies that no two of the action rules can be satisfied simultaneously.

There are six different pairs of the four rules on page 25. Only two of these, namely, {east, west} and {south, north} have conditions that are not contradictory. For the conditions on east and west both to be satisfied, we would need $x_1 = 1$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 0$, but that would violate the tight–space restriction. Similarly, for the conditions on south and north both to be satisfied, we would need $x_2 = 1$, $x_3 = 0$, $x_4 = 1$, and $x_1 = 0$, and that would also violate the tight–space restriction. (If all x_i are 0, none of the four rules is satisfied, and the robot moves north.)

2.6 Design (by hand) a neural network that accepts as inputs the sensory signals s_1, s_2,..., s_8 and produces as outputs the conditions needed by a network of TISA units to implement the action rules on page 25 for the wall-following robot.

The production rules to be used by the TISA network are given on page 28. As shown, the conditions needed as input to the TISA network are:

$x_4\overline{x_1}$

$x_3\overline{x_4}$

$x_2\overline{x_3}$

$x_1\overline{x_2}$

1

(The last of these, 1, is trivial to implement.)

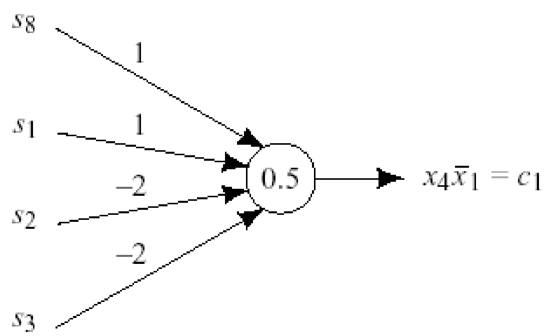
It was shown in Figure 2.6 (page 30) how to implement the condition $x_1\overline{x_2} = (s_2 + s_3)\overline{s_4}\,\overline{s_5}$ by a TLU. Expressing the other conditions in terms of the s_i yields:

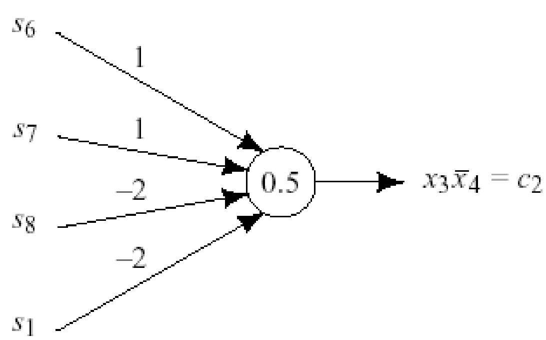
$$x_4\overline{x_1} = (s_8 + s_1)\overline{s_2}\,\overline{s_2}$$

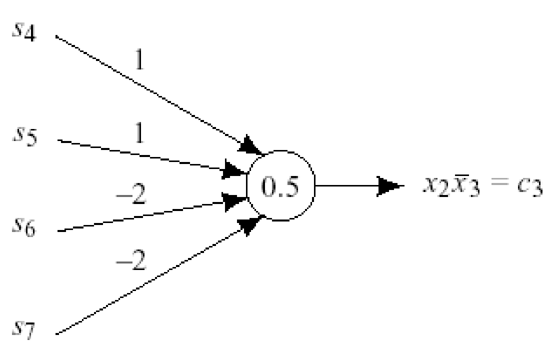
$$x_3\overline{x_4} = (s_6 + s_7)\overline{s_8}\,\overline{s_1}$$

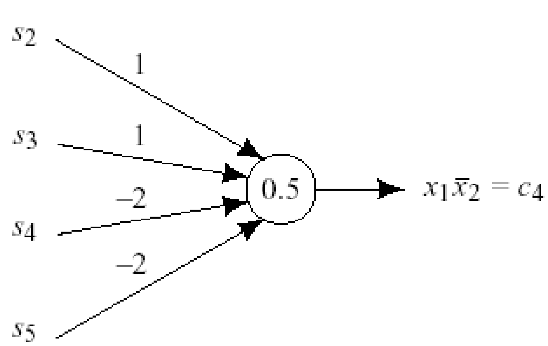
$$x_2\overline{x_3} = (s_4 + s_5)\overline{s_6}\,\overline{s_7}$$

The TLU implementations of these three conditions are similar to the one shown in Figure 2.6. The TLUs needed by a network to implement all four of these conditions are shown on the next page.


s_8
1
s_1
1
-2
s_2
-2
s_3
0.5
$x_4\bar{x}_1 = c_1$


s_6
1
s_7
1
-2
s_8
-2
s_1
0.5
$x_3\bar{x}_4 = c_2$


s_4
1
s_5
1
-2
s_6
-2
s_7
0.5
$x_2\bar{x}_3 = c_3$


s_2
1
s_3
1
-2
s_4
-2
s_5
0.5
$x_1\bar{x}_2 = c_4$