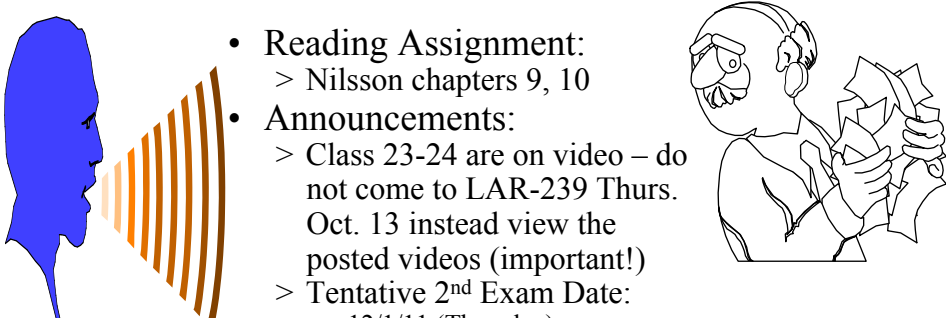


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EEL5840: Elements of Machine Intelligence

Announcements

- Reading Assignment:
 - > Nilsson chapters 9, 10
- Announcements:
 - > Class 23-24 are on video – do not come to LAR-239 Thurs. Oct. 13 instead view the posted videos (important!)
 - > Tentative 2nd Exam Date:
 - 12/1/11 (Thursday)
 - > LISP Project due 11/29/11
- Today's Handouts in WWW:
 - > Outline Class 22
- Web Site
 - > www.mil.ufl.edu/eel5840
 - > Software and Notes



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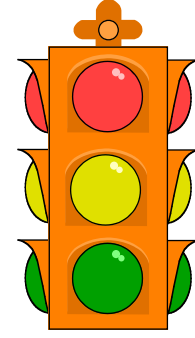
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Today's Menu

- Slides 5-8 in Class 21
- Heuristic Search (Chapter 9) (continued)
 - ⇒ Admissibility of A*



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Admissibility of A*

Admissible : An algorithm is admissible if for any graph it *always terminates* in an optimal path from $s \rightarrow t$ if such a path exists.
Admissibility implies GRAPH-SEARCH must terminate!

Result 1 *GRAPH-SEARCH always terminates for finite graphs*

Proof: In every cycle of the algorithm we remove a node from OPEN and only a finite number of new successors are added. Since the graph is finite, we ultimately run out of new successors and we will either terminate in step 5 by finding a goal or in step 3 by running out of nodes.



Can we show that A* terminates even for infinite graphs if a path from $s \rightarrow t$ exists?

Suppose A* does not terminate, that is we never quit adding nodes to OPEN. Then even the smallest $f(n)$ will eventually grow to ∞ .

Why? $f(n) = g(n) + h(n)$ & since $g(n)$ is a depth component
 $g(n) \rightarrow \infty$ as $n \rightarrow \infty$ {infinite graphs have infinite depth}



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Admissibility of A*

If that happens, then a goal node t which is sitting in OPEN will eventually migrate to the front of the list where step 5 will take the *car/first* of the list, remove it and terminate. Note that for a goal node t , $f(t) = g(t) + h(t) = g(t)$ and $h(t) = 0$ { $k(t,t) = cost(t,t) = 0$ by definition} and $g(t)$ is a finite number (this is the cost of the path $s \rightarrow t$ we assumed exists).

Mathematically

Recall by definition $c(n_i, n_j) \geq e > 0$ (e is a small positive number) and step 7 of GRAPH-SEARCH guarantees that $g(n) \geq g^*(n)$

or $g^*(n) \leq g(n)$ and $0 \leq h(n) \leq h^*(n)$ in A*

Let $d^*(n)$: length of the shortest path $s \rightarrow n$

$\therefore g(n) \geq g^*(n) \geq d^*(n)e$ and since $f(n) = g(n) + h(n)$ and $h(n) \geq 0$
 then $f(n) \geq g(n) \geq d^*(n)e$

{every node n on OPEN is at least as large as $d^*(n)e$ }



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Admissibility of A^*

But if A^* does not terminate and we never quit adding nodes to OPEN, then $f(n) \rightarrow \infty$ since $d(n) \rightarrow \infty$ as $n \rightarrow \infty$. These are large f -value nodes added to the existing nodes in OPEN.

To show A^* terminates we will now show that there is always one node n' on OPEN that has a finite value given by $f(n) \leq f^*(s)$

Let a path $s \rightarrow n_k$ be optimal and ordered, that is
 $\text{path}^*(s \rightarrow n_k) = \{s = n_0, n_1, n_2, \dots, n_{k-1}, n_k\}$

Before termination let n' be the 1st node on OPEN that is a member of the path $^*(s \rightarrow n_k)$.

Q: Is there such a member n' ? A: Yes! Why? To start with $s = n_0$ is a member of OPEN and after we enter the loop n_k cannot be a member of CLOSED (else we've terminated!)



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Algorithm A^*

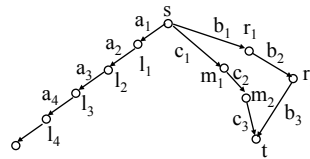
Example:

Let $a's = \{0.5\}$ $b's = \{1.2, 1, 1.5\}$

$c's = \{1, 0.5, 0.75\}$

Initially: OPEN = {s}

CLOSED = {} ∞



1st Pass: Expand s,

$M = \Gamma(s) = \{l_1, m_1, r_1\}$

OPEN = $\{l_1, m_1, r_1\}$

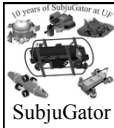
$G = \{s, l_1, m_1, r_1\}$

Pointers = $\{\text{nil}, s, s, s\}$

CLOSED = $\{s\}$

$\{0.5, 1, 1.2\}$

\therefore Expand l_1



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Admissibility of A*

2nd Pass: Expand l_1 , $M=\Gamma(l_1)=\{l_2\}$ CLOSED= $\{s, l_1\}$

$G=\{s, l_1, m_1, r_1, l_2\}$ Pointers= $\{\text{nil}, s, s, s, l_1\}$

OPEN= $\{l_2, m_1, r_1\}$ F= $\{1, 1, 1.2\}$ ∴ Expand l_2

3rd Pass: Expand l_2 , $M=\Gamma(l_2)=\{l_3\}$ CLOSED = $\{s, l_1, l_2\}$

$G=\{s, l_1, m_1, r_1, l_2, l_3\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2\}$

OPEN = $\{m_1, r_1, l_3\}$ F= $\{1, 1.2, 1.5\}$ ∴ Expand m_1

4th Pass: Expand m_1 , $M=\Gamma(m_1)=\{m_2\}$ CLOSED = $\{s, l_1, l_2, m_1\}$

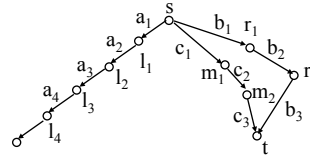
$G=\{s, l_1, m_1, r_1, l_2, l_3, m_2\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2, m_1\}$

OPEN = $\{r_1, m_2, l_3\}$ F= $\{1.2, 1.5, 1.5\}$ ∴ Expand r_1

5th Pass: Expand r_1 , $M=\Gamma(r_1)=\{r_2\}$ CLOSED = $\{s, l_1, l_2, m_1, r_1\}$

$G=\{s, l_1, m_1, r_1, l_2, l_3, m_2, r_2\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2, m_1, r_1\}$

OPEN = $\{m_2, l_3, r_2\}$ F= $\{1.5, 1.5, 2.2\}$ ∴ Expand m_2



NOTE: At all times in OPEN there is a node n' , which is a member of the optimal path*= $\{s, m_1, m_2, t\}$. Also note that the f value of the nodes in path* is $f^*(s) = c_1 + c_2 + c_3$



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Admissibility of A*

6th Pass: Expand m_2 , $M=\Gamma(m_2)=\{t\}$ CLOSED = $\{s, l_1, l_2, m_1, r_1, m_2\}$

$G=\{s, l_1, m_1, r_1, l_2, l_3, m_2, r_2, t\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2, m_1, r_1, m_2\}$

OPEN = $\{l_3, r_2, t\}$ F= $\{1.5, 2.2, 2.25\}$ ∴ Expand l_3

7th Pass: Expand l_3 , $M=\Gamma(l_3)=\{l_4\}$ CLOSED = $\{s, l_1, l_2, m_1, r_1, m_2, l_3\}$

$G=\{s, l_1, m_1, r_1, l_2, l_3, m_2, r_2, t, l_4\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2, m_1, r_1, m_2, l_3\}$

OPEN = $\{l_4, r_2, t\}$ F= $\{2, 2.2, 2.25\}$ ∴ Expand l_4

8th Pass: Expand l_4 , $M=\Gamma(l_4)=\{l_5\}$ CLOSED = $\{s, l_1, l_2, m_1, r_1, m_2, l_3, l_4\}$

$G=\{s, l_1, m_1, r_1, l_2, l_3, m_2, r_2, t, l_4, l_5\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2, m_1, r_1, m_2, l_3, l_4\}$

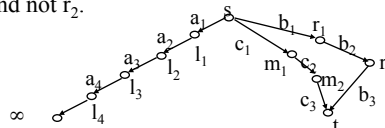
OPEN = $\{r_2, t, l_5\}$ F= $\{2.2, 2.25, 2.5\}$ ∴ Expand r_2

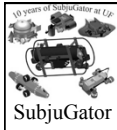
9th Pass: Expand r_2 , $M=\Gamma(r_2)=\{t\}$ CLOSED = $\{s, l_1, l_2, m_1, r_1, m_2, l_3, l_4, r_2\}$

$G=\{s, l_1, m_1, r_1, l_2, l_3, m_2, r_2, t, l_4, l_5\}$ Pointers= $\{\text{nil}, s, s, s, l_1, l_2, m_1, r_1, m_2, l_3, l_4\}$

OPEN = $\{t, l_5\}$ F= $\{2.25, 2.5\}$ ∴ Success! Found t Path is $t \rightarrow m_2 \rightarrow m_1 \rightarrow s$

NOTE: In step 9 above when $n=t$ is found as a successor of r_2 step 7 of the algorithm checks and finds that $n=t$ is already in OPEN and keeps the shortest path $\{the\ cost\ c_1 + c_2 + c_3 < b_1 + b_2 + b_3\}$ to $n=t$ which is still through m_2 and not r_2 .





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The End!