



EEL5840: Elements of Machine Intelligence

SubjuGator

Announcements



- Reading Assignment:
 - > Nilsson chapters 9, 10
- Announcements:
 - > Tentative 2nd Exam Dates:
 - 12/1/09 (Tuesday)
 - 12/3/09 (Thursday)
- Today's Handouts in WWW:
 - > Outline Class 22
- Web Site
 - > www.mil.ufl.edu/eel5840
 - > Software and Notes



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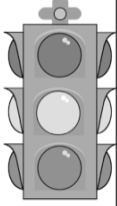
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Today's Menu

- Heuristic Search (Chapter 9) (continued)
 - ⇒ Admissibility of A*



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Admissibility of A*

Admissible : An algorithm is admissible if for any graph it *always terminates* in an optimal path from $s \rightarrow t$ if such a path exists.
Admissibility implies GRAPH-SEARCH must terminate!
 Result 1 *GRAPH-SEARCH always terminates for finite graphs*
 Proof: In every cycle of the algorithm we remove a node from OPEN and only a finite number of new successors are added. Since the graph is finite, we ultimately run out of new successors and we will either terminate in step 5 by finding a goal or in step 3 by running out of nodes.

◆ Can we show that A* terminates even for infinite graphs if a path from $s \rightarrow t$ exists?
 Suppose A* does not terminate, that is we never quit adding nodes to OPEN. Then even the smallest $f(n)$ will eventually grow to ∞ .
 Why? $f(n) = g(n) + h(n)$ & since $g(n)$ is a depth component $g(n) \rightarrow \infty$ as $n \rightarrow \infty$ (infinite graphs have infinite depth)

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If that happens, then a goal node t which is sitting in OPEN will eventually migrate to the front of the list where step 5 will take the *car* of the list, remove it and terminate. Note that for a goal node t , $f(t) = g(t) + h(t) = g(t)$ and $h(t) = 0$ $\{k(t,t) = cost(t,t) = 0$ by definition} and $g(t)$ is a finite number (this is the cost of the path $s \rightarrow t$ we assumed exists).

Mathematically
 Recall by definition $c(n_p, n_j) \geq e > 0$ (e is a small positive number) and step 7 of GRAPH-SEARCH guarantees that $g(n) \geq g^*(n)$ or $g^*(n) \leq g(n)$ and $0 \leq h(n) \leq h^*(n)$ in A*
 Let $d^*(n)$: length of the shortest path $s \rightarrow n$
 $\therefore g(n) \geq g^*(n) \geq d^*(n)e$ and since $f(n) = g(n) + h(n)$ and $h(n) \geq 0$ then $f(n) \geq g(n) \geq d^*(n)e$
 {every node n on OPEN is at least as large as $d^*(n)e$ }

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But if A* does not terminate and we never quit adding nodes to OPEN, then $f(n) \rightarrow \infty$ since $d(n) \rightarrow \infty$ as $n \rightarrow \infty$. These are large f -value nodes added to the existing nodes in OPEN.

To show A* terminates we will now show that there is always one node n' on OPEN that has a finite value given by $f(n) \leq f^*(s)$

Let a path $s \rightarrow n_k$ be optimal and ordered, that is
 $path^*(s \rightarrow n_k) = \{s = n_0, n_1, n_2, \dots, n_{k-1}, n_k\}$

Before termination let n' be the 1st node on OPEN that is a member of the path $^*(s \rightarrow n_k)$.

Q: Is there such a member n' ? A: Yes! Why? To start with $s = n_0$ is a member of OPEN and after we enter the loop n_k cannot be a member of CLOSED (else we've terminated!)

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Algorithm A*

Example:

Let $a's = \{0.5\}$ $b's = \{1.2, 1, 1.5\}$
 $c's = \{1, 0.5, 0.75\}$
 Initially: OPEN = {s}
 CLOSED = {} ∞

1st Pass: Expand s,
 $M = \Gamma(s) = \{l_1, m_1, r_1\}$
 OPEN = $\{l_1, m_1, r_1\}$
 CLOSED = {s}
 $\{0.5, 1, 1.2\}$
 \therefore Expand l_1

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2nd Pass: Expand l_1 , $M = \Gamma(l_1) = \{l_2\}$
 OPEN = $\{l_2, m_1, r_1\}$ CLOSED = $\{s, l_1\}$
 $\{1, 1, 1.2\} \therefore$ Expand l_2

3rd Pass: Expand l_2 , $M = \Gamma(l_2) = \{l_3\}$
 OPEN = $\{m_1, r_1, l_3\}$ CLOSED = $\{s, l_1, l_2\}$
 $\{1, 1.2, 1.5\} \therefore$ Expand m_1

4th Pass: Expand m_1 , $M = \Gamma(m_1) = \{m_2\}$
 OPEN = $\{r_1, m_2, l_3\}$ CLOSED = $\{s, l_1, l_2, m_1\}$
 $\{1.2, 1.5, 1.5\} \therefore$ Expand r_1

5th Pass: Expand r_1 , $M = \Gamma(r_1) = \{r_2\}$
 OPEN = $\{m_2, l_3, r_2\}$ CLOSED = $\{s, l_1, l_2, m_1, r_1\}$
 $\{1.5, 1.5, 2.2\} \therefore$ Expand m_2

NOTE: At all times in OPEN there is a node n' , which is a member of the optimal path $^* = \{s, m_1, m_2, t\}$. Also note that the f value of the nodes in path* is $f^*(s) = c_1 + c_2 + c_3$

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Admissibility of A*

WHAT DO WE KNOW ABOUT n' ?
 $f(n') = g(n') + h(n')$ and $g(n') = g^*(n')$ {since n' is on the optimal path} and further $0 \leq h(n) \leq h^*(n) \forall n$

Thus,
 $f(n') = g^*(n') + h(n')$ and $0 \leq h(n') \leq h^*(n')$
 $\therefore f(n') \leq g^*(n') + h^*(n') = f^*(n')$

BUT, the f value of any node n' in an optimal path is finite and is given by $f^*(s)$.

Why? Consider the following
 $f^*(s) = g^*(s) + h^*(s) = h^*(s)$ {since $g^*(s) = 0$ by definition}
 $f^*(s) = g^*(n') + h^*(n') = f^*(n')$ {for any node n' }

Therefore, $f(n') \leq f^*(n') = f^*(s)$

RESULT 2 At any time before termination A* has a node n' that is a member of the optimal path $^*(s \rightarrow n_k)$ such that $f(n') \leq f^*(s)$

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WE ARE ALL SET:

- Assume A* does not terminate, so $f(n) \rightarrow \infty$
- There is a node n' on OPEN that is a member of the optimal path* ($s \rightarrow n_k$) such that $f(n') \leq f^*(s)$ which is a finite value.
- When we order in step 8 according to f values, node n' will migrate to the front because its f value $< \infty$
- \therefore A* will terminate (which causes a contradiction)

◆

RESULT 3 If an optimal path exists then A* will terminate

Corollary 3 Any node n with $f(n) < f^*(s)$ will eventually be selected for expansion. Why? Since any node n' on the optimal path* ($s \rightarrow n_k$) was selected with $f(n') \leq f^*(s)$ when you terminated then the node with $f(n) < f^*(s)$ had to also be selected.

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Admissibility of A*

To prove that A* only finds optimal paths at termination

Let A* terminate at node t but without finding an optimal path

Then, $f(t) = g(t) + h(t)$ $h(t)=0$ and for n' $f(n') = f^*(s)$
 $f(t) = g(t) > f^*(s)$ {A* terminated at node t w/o optimality}

However, by RESULT 2 there is a node n' on the optimal path* with $f(n') \leq f^*(s) < f(t)$

But according to step 8, A* would order the nodes such that n' is picked before t , n' precedes t , i.e., A* could not terminate. This contradicts the assumption \therefore

◆

◆ RESULT 4 A* is admissible (if there is a path* ($s \rightarrow t$) A* finds it)

◆

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RESULT 5 For any node n selected for expansion $f(n) \leq f^*(s)$

Proof: If $n=s$ then $f(s)=f^*(s) \leq f^*(s)$ is trivially true
 If $n=t$ then $f(t)=f^*(s) \leq f^*(s)$ is trivially true
 If $n=n'$ then $f(n') \leq f^*(s)$ by RESULT 2
 If $n \neq n'$ then to select n it must be that $f(n) \leq f(n')$
 thus, $f(n) \leq f(n')$ and $f(n') \leq f^*(s)$ or $f(n) \leq f^*(s)$ Q.E.D.

RESULT 6 If A*₂ is more informed than A*₁ ($0 \leq h_1(n) \leq h_2(n) \leq h^*(n)$) then at termination, A*₁ expands at least as many nodes as A*₂ or every node expanded by A*₂ is expanded by A*₁

Proof: A proof by induction assumes the trivial $m=0$ case and proves the m^{th} case given the $(m-1)^{\text{th}}$ case holds.

Step 1: Both expand $n=s$ for $m=\text{depth}(n)=0$. trivial case holds

Step 2: Assume it holds for $m-1$ depth and show the m^{th} case i.e., Up to depth $m-1$ nodes_in(A*₁) \geq nodes_in(A*₂)

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We note that every ancestor of node n is in both A*₁ and A*₂ since they are both working on the same problem and we assumed that up to depth $m-1$ they both have expanded the same nodes, so $g_1(n) \leq g_2(n)$. Now we assume node n is expanded by A*₂ and not by A*₁ and we let them both terminate. Node n must be in OPEN₁ but in CLOSED₂. And $f^*(s)$ is the same in both graphs.

In A*₁ we have $f_1(n) \geq f^*(s)$ or $g_1(n)+h_1(n) \geq f^*(s)$ or $h_1(n) \geq f^*(s)-g_1(n)$
 In A*₂ we have $f_2(n) \leq f^*(s)$ or $g_2(n)+h_2(n) \leq f^*(s)$ or $h_2(n) \leq f^*(s)-g_2(n)$

With $g_1(n) \leq g_2(n)$ then $h_1(n) \geq f^*(s)-g_1(n) >> f^*(s)-g_2(n)$ and now $h_2(n) < h_1(n)$. But we assumed that $0 \leq h_1(n) \leq h_2(n)$ so we have a contradiction, thus, n which was expanded by A*₂ must also have been expanded by A*₁ \therefore RESULT 6 is true!

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Admissibility of A*

2 8 3
1 6 4 $W(s)=4$

$h_1(n)=W(n)$ = number of tiles out of place
{10 total nodes expanded with $W(n)$ }

2 8 3
1 6 4 $P(s)=5$

$h_2(n)=P(n)$ =sum of distances from destination, i.e., Manhattan Distance
{8 total nodes expanded with $P(n)$ }

Therefore $P(n)$ is more informed than $W(n)$

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Definition: Monotone cost functions
If $h(n)$ is a heuristic function such that $h(n)=0$ and further $0 < h(n_i)-h(n_j) \leq \text{cost}(n_i, n_j)$ then $h(n)$ is a monotone cost function.

RESULT 7 If $h(n)$ is a monotone cost function then A* does not ever need to redirect pointers (in Step 7) and $g(n)=g^*(n)$

Proof Let path* = { $s=n_0, n_1, n_2, \dots, n_i, n_{i+1}, \dots, n_k, n_k=n$ } and n_i be on CLOSED and n_{i+1} be on OPEN. Also $g(n_i)=g^*(n_i)$
 $\forall n_i$ on path* then $g(n_i)+h(n_i) \leq g(n_i)+h(n_i)+\text{cost}(n_i, n_{i+1})$
and let $j=i+1$ on path* then
 $g(n_i)+h(n_i) \leq g(n_i)+h(n_{i+1})+\text{cost}(n_i, n_{i+1})$ or
 $g^*(n_i)+h(n_i) \leq g^*(n_i)+h(n_{i+1})+\text{cost}(n_i, n_{i+1})$ and
 $\text{cost}(n_i, n_{i+1}) = g^*(n_{i+1}) - g^*(n_i)$ thus
 $g^*(n_i)+h(n_i) \leq g^*(n_{i+1})+h(n_{i+1})$

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Since $g^*(n_i)+h(n_i) \leq g^*(n_{i+1})+h(n_{i+1})$ let $i-l+1=k-1$ and $i+1=k$
 $g^*(n_{i+1})+h(n_{i+1}) \leq g^*(n_k)+h(n_k)$ or $f(n_{i+1}) \leq f(n_k) = f(n)$
if A* selects node n over node n_{i+1} it must be
 $f(n) = g(n) + h(n) \leq f(n_{i+1})$ then $g(n) \leq g^*(n)$
and since Graph-Search guarantees that $g^*(n) \leq g(n)$
then $g(n)=g^*(n)$ Q.E.D.

RESULT 8 If $h(n)$ is a monotone cost function then the f values of a sequence of nodes is non-decreasing in A*

Proof Let n_2 be expanded after n_1 . If n_2 was on OPEN then $f(n_1) \leq f(n_2)$ trivially, so n_2 was not on OPEN, i.e., $n_2 \in \Gamma(n_1)$
 $f(n_2) = g(n_2) + h(n_2)$ and since $h(n)$ is monotonic then $0 < h(n_1)-h(n_2) \leq \text{cost}(n_1, n_2)$ and by RESULT 7
 $g(n_2) = g^*(n_2)$ and by definition $g^*(n_2) = g^*(n_1) + \text{cost}(n_1, n_2)$ thus
 $f(n_2) = g(n_2) + h(n_2) = g^*(n_2) + h(n_2) = g^*(n_1) + \text{cost}(n_1, n_2) + h(n_2)$
or $f(n_2) \geq g^*(n_1) + h(n_1) = f(n_1)$ Q.E.D.

• What happens if $h(n) > h^*(n)$, then we have no A* and the path found may not be optimal. Yet, it may still work in some cases!

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Admissibility of A*

Proof (Continued)
 $f(n_2) = g(n_2) + h(n_2)$ and since $h(n)$ is monotonic then $0 < h(n_1)-h(n_2) \leq \text{cost}(n_1, n_2)$ and by RESULT 7
 $g(n_2) = g^*(n_2)$ and by definition
 $g^*(n_2) = g^*(n_1) + \text{cost}(n_1, n_2)$ thus
 $f(n_2) = g(n_2) + h(n_2) = g^*(n_2) + h(n_2) = g^*(n_1) + \text{cost}(n_1, n_2) + h(n_2)$
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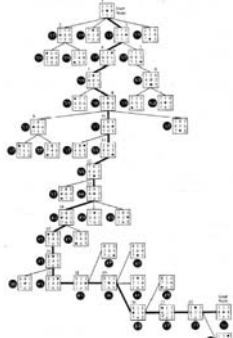
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- Here $h_3(n) = P(n) + 3S(n)$, where $S(n)$ is a sequence score obtained by checking around non-central squares in turn, allotting 2 for every tile not followed by its proper successor and allotting 0 for every other tile; a tile in the center scores 1.

Q1: Is this $h_3(n)$ a lower bound of $h^*(n)$?
Q2: Is $h_4(n) = S(n)$ a lower bound of $h^*(n)$?



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Heuristic Searches

Factors to Consider in Heuristic Searches

- $h(n)$ may be very complex and costly
- Let $f(n) = \alpha g(n) + \beta h(n)$ $\alpha, \beta \geq 0$ α controls the Breadth-First component and β controls the Depth-First component
- To insure some path to a goal will be found, one must include $g(n)$ even if finding a minimal-cost path is not necessary
- β should vary inversely proportional to depth. At shallow depth, increase β and at deeper nodes increase α
- Pay attention to
 - > the total cost of a path
 - > the number of nodes expanded in finding the path
 - > the computational effort to compute $h(n)$
- Seek for balance in your approach

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The End!

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