



EEL5840: Elements of Machine Intelligence

Robobug

## Announcements



- Reading Assignment:
  - > Nilsson chapter 9
- Announcements:
  - > Tentative 2<sup>nd</sup> Exam Dates:
    - 12/1/09 (Tuesday)
    - 12/3/09 (Thursday)
- Today's Handouts in WWW:
  - > Outline Class 21
- Web Site
  - > [www.mil.ufl.edu/eel5840](http://www.mil.ufl.edu/eel5840)
  - > Software and Notes



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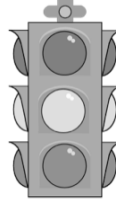
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## Today's Menu

- Heuristic Search (Chapter 9)
  - ⇒ Algorithm A\*
  - ⇒ Admissibility of A\*



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## Search Strategies

### PROCEDURE GRAPH-SEARCH

1. Create a *search graph*,  $G$ , consisting solely of the start node,  $s$ . Put  $s$  on a list called *OPEN*.
2. Create a list called *CLOSED* that is initially empty.
3. LOOP: if *OPEN* is empty, exit with failure.
4. Select the first node on *OPEN*, remove it from *OPEN*, and put it on *CLOSED*. Call this node  $n$ .
5. If  $n$  is a goal node, exit successfully with the solution obtained by tracing a path along the pointers from  $n$  to  $s$  in  $G$ . (see step 7.)
6. Expand node  $n$ , generating the set,  $M$ , of its successors and install them as successors of  $n$  in  $G$ .

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## Search Strategies

7. Establish a pointer to  $n$  from those members of  $M$  that were not already in  $G$  (i.e., not already on either *OPEN* or *CLOSED*). Add these members of  $M$  to *OPEN*. For each member of  $M$  that was already on *OPEN* or *CLOSED*, decide whether or not to redirect its pointer to  $n$ . For each member of  $M$  already on *CLOSED*, decide for each of its descendants in  $G$  whether or not to redirect its pointer.
8. Reorder the list *OPEN*, either according to some arbitrary scheme or according to heuristic merit.
9. GO LOOP

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## Algorithm A

**ALGORITHM A**  
 Let  $f(n) = g(n) + h(n)$  in step 8 of GRAPH-SEARCH where  
 $g(n)$  : estimate of the cost of a minimum length path  $s \rightarrow n$   
 $h(n)$  : estimate of the cost of a minimum length path  $n \rightarrow t$   
 also, step 7 guarantees that  $g(n)$  can never increase.  
 {See slides 6-7 in class 21}

In the example:  
 $c(n_1, n_2) = \text{cost from } n_1 \rightarrow n_2$   
 $g(n) = c(n_1, n) = c(n_1, n_2) + c(n_2, n) = c_1 + c_2$   
 $h^*(n) = c(n, t) = c_3$  (the actual but unknown cost from  $n \rightarrow t$ )  
 $h(n)$  is an estimate of  $h^*(n)$ . When we are at node  $n$  we have not finished the problem and we do not yet have the real  $c(n, t)$ . We say that  $f(n)$  is the cost of a minimal cost path constrained through node  $n$ . Here,  $c_6 < c_3$  and  $c_3 \leq c_6 + c_7$  by the triangle inequality.

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## Algorithm A

**DEFINITIONS**

- $f(n)$  : estimate of the cost of a path  $s \rightarrow t$  through node  $n$   
 $c(s, t) = c(s, n) + c(n, t)$
- $k(n, n_j)$  : actual cost of a minimal-cost path  $n_1 \rightarrow n_j$
- $h^*(n)$  : cost of a minimal cost path from node  $n$  to a goal, i.e.,  $\min(k(n, t_1), k(n, t_2), \dots, k(n, t_i))$  and any node that achieves  $h^*(n)$  is a node in the optimal path.
- $g^*(n)$  : cost of an optimal path from  $s \rightarrow n$  (the shortest path  $s \rightarrow n$ )  
 $= k(s, n) \forall n$  accessible from  $s$
- $f^*(n)$  : (actual cost of an optimal path  $s \rightarrow n$ ) + (cost of an optimal path  $n \rightarrow t$ ) {the path  $s \rightarrow t$  is constrained to go through  $n$ }
- $f(n) = g(n) + h(n) \forall n$  and  $f^*(n) = g^*(n) + h^*(n) \forall n$

Now when  $n = s$ ;  
 $f^*(s) = g^*(s) + h^*(s) = h^*(s)$  since by definition  $g^*(s) = 0 = k(s, s)$   
 $h^*(s)$  : actual cost of an unconstrained path  $s \rightarrow t$

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## Algorithm A

Let  $g(n)$  be an estimate of  $g^*(n)$  and  $h(n)$  be an estimate of  $h^*(n)$ .  
 Then  $f(n)$  will be an estimate of  $f^*(n)$  and further let  
 $E\{f(n) - f^*(n)\} = 0$  and  
 Variance $\{f(n)\}$  be non-increasing

We say that  $f(n)$  is a consistent estimate of  $f^*(n)$  and we expect algorithm A to yield good results.

Q: Are there any properties of  $f$ ,  $g$ , or  $h$  that insure optimal results?

We also note :  
 if  $h(n) = 0$  and  $g(n) = \text{depth}(n)$  algorithm A yields BFS  
 and if  $g(n) = 0$  algorithm A yields pure heuristic search (DFS)  
 $f(n) = \alpha g(n) + \beta h(n) = \alpha \{\text{breadth component}\} + \beta \{\text{depth component}\}$   
 and  $\{\alpha, \beta\} = 1$  in Algorithm A but it can be more general

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## Algorithm A\*

**ALGORITHM A\***  
 Let  $f(n) = g(n) + h(n)$  in step 8 of GRAPH-SEARCH where  
 $g(n)$  : estimate of the cost of a minimum length path  $s \rightarrow n$   
 $h(n)$  : estimate of the cost of a minimum length path  $n \rightarrow t$   
 and  $0 \leq h(n) \leq h^*(n)$  and step 7 guarantees that  $g^*(n) \leq g(n)$

- Algorithm A\* always finds the optimal path from  $s \rightarrow t$
- Notice that  $h(n) \leq h^*(n)$  means that  $h(n)$  underestimates the actual optimal cost  $h^*(n)$  and thus,  $h(n)$  is a conservative estimate of  $h^*(n)$
- Since  $h(n) = 0$  is an underestimate of  $h^*(n)$  then A\* with  $g(n) = \text{depth}(n)$  always finds the optimal path  $s \rightarrow t$ . That is, BFS, is optimal.
- On the other hand  $g(n) = 0$ , that is heuristic search (DFS), does not carry the same guarantee as we will see later.

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*The End!*

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